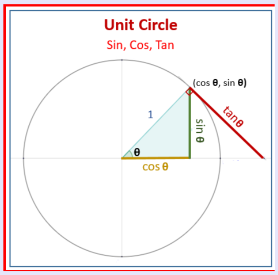
Trigonometry

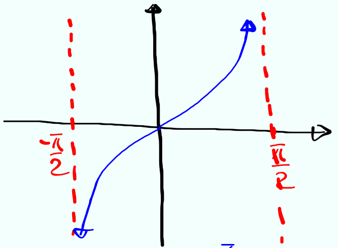
Lecture 42



Feb 19-8:47 AM

More on inverse functions

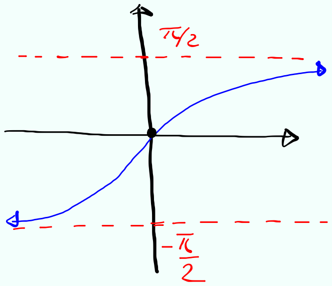
$y = \tan x$



$-\frac{\pi}{2} < x < \frac{\pi}{2}$

$-\infty < y < \infty$

$y = \tan^{-1} x$



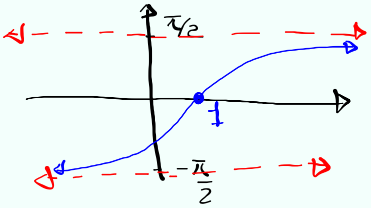
$-\infty < x < \infty$

$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Graph $y = \tan^{-1}(x-1)$

$x - 1 = 0$

$x = 1$

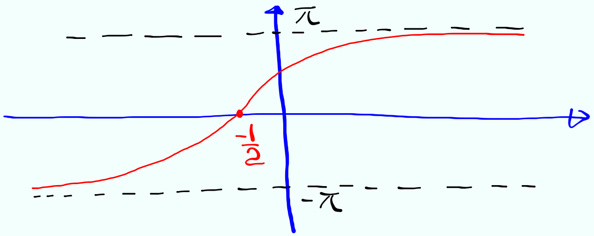


Nov 14-10:26 AM

Graph $y = 2\tan^{-1}(2x+1)$

$2x+1=0$

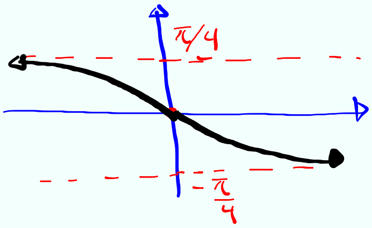
$x = \frac{-1}{2}$



Graph $y = -\frac{1}{2}\tan^{-1}\frac{1}{2}x$

$\frac{1}{2}x = 0$

$x = 0$



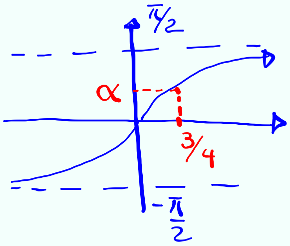
Nov 14-10:31 AM

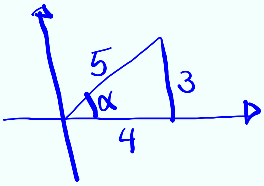
Find exact value of $\tan\left(2\tan^{-1}\frac{3}{4}\right)$

$\alpha = \tan^{-1}\frac{3}{4}$

$\tan\alpha = \frac{3}{4}$

$0 < \alpha < \frac{\pi}{2}$





$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}}$$

LCD = 16

$$= \frac{16\cdot\frac{3}{2}}{16\cdot 1 - 16\cdot\frac{9}{16}} = \boxed{\frac{24}{7}}$$

Use Your Calc

$\tan(2\cdot\tan^{-1}.75)$ ≈ 3.429

Nov 14-10:36 AM

Find exact value of $\tan\left(\frac{1}{2}\tan^{-1}\frac{-3}{4}\right)$

$\alpha = \tan^{-1}\frac{-3}{4}$

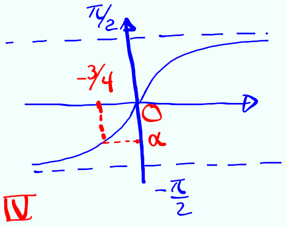
$\tan\alpha = \frac{-3}{4}$

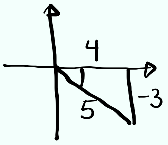
α is in Q IV

$-90^\circ < \alpha < 0^\circ$

$-45^\circ < \frac{1}{2}\alpha < 0^\circ$

Q IV





$\tan\left(\frac{1}{2}\alpha\right)$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$$

$$= \frac{1-\frac{4}{5}}{\frac{-3}{5}}$$

LCD = 5

$$= \frac{5-4}{-3} = \boxed{\frac{-1}{3}}$$

Use your calc.

$\tan\left(.5 \cdot \tan^{-1}(-.75)\right)$

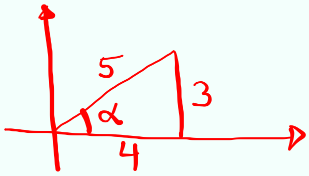
$= -.\overline{3} = -\frac{1}{3}$

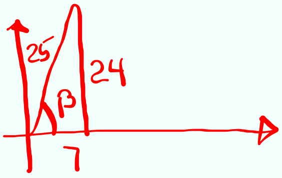
Nov 14-10:42 AM

Find exact value of $\tan\left(\underbrace{\sin^{-1}\frac{3}{5}}_{\alpha} - \underbrace{\cos^{-1}\frac{7}{25}}_{\beta}\right)$

$\alpha = \sin^{-1}\frac{3}{5}$ $\beta = \cos^{-1}\frac{7}{25}$

$\sin\alpha = \frac{3}{5}$ $\cos\beta = \frac{7}{25}$





$\tan(\alpha - \beta)$

$$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$= \frac{\frac{3}{4} - \frac{24}{7}}{1 + \frac{3}{4}\cdot\frac{24}{7}}$$

LCD = 28

$$\frac{\overset{7}{\cancel{28}}\cdot\frac{3}{\cancel{4}} - \overset{4}{\cancel{28}}\cdot\frac{24}{\cancel{7}}}{28\cdot 1 + \cancel{28}\cdot\frac{3}{\cancel{4}}\cdot\frac{24}{\cancel{7}}} = \frac{21-96}{28+72} = \frac{-75}{100}$$

$$= \boxed{\frac{-3}{4}}$$

Nov 14-10:48 AM

Solve $\sqrt{2}\,\mathrm{Cos}\,3x - 1 = 0$ on $[0^\circ, 360^\circ)$.

$\mathrm{Cos}\,3x = \frac{1}{\sqrt{2}}$ QI $3x = 45^\circ + K\cdot 360^\circ$

$\mathrm{Cos}\,3x = \frac{\sqrt{2}}{2}$ $\boxed{x = 15^\circ + K\cdot 120^\circ}$

QI & QIV

RA. 45°

QIV $3x = 360^\circ - 45^\circ + K\cdot 360^\circ$

$\boxed{x = 105^\circ + K\cdot 120^\circ}$

K=0 $x = 15^\circ, 105^\circ$

K=1 $x = 135^\circ, 225^\circ$

K=2 $x = 255^\circ, 345^\circ$

~~K=3~~ ~~$15^\circ + 360^\circ$~~

~~$105^\circ + 360^\circ$~~

Nov 14-10:55 AM

Find all general solutions on $[0, 2\pi)$ for

$2\mathrm{Sin}\,3x + 1 = 0.$

$\mathrm{Sin}\,3x = \frac{-1}{2}$

QIII & QIV

RA $30^\circ = \frac{\pi}{6}$

QIII $3x = \pi + \frac{\pi}{6} + K\cdot 2\pi$

$3x = \frac{7\pi}{6} + K\cdot 2\pi \rightarrow x = \frac{7\pi}{18} + \frac{K\cdot 2\pi}{3}$

QIV $3x = 2\pi - \frac{\pi}{6} + K\cdot 2\pi$

$3x = \frac{11\pi}{6} + K\cdot 2\pi \rightarrow x = \frac{11\pi}{18} + \frac{K\cdot 2\pi}{3}$

Nov 14-11:02 AM

Write $\underset{\uparrow A}{\sin x} + \underset{\uparrow B}{\cos x}$ as $K\sin(x+\alpha)$

where $K=\sqrt{A^2+B^2}$ $\cos\alpha=\frac{A}{K}$ & $\sin\alpha=\frac{B}{K}$

$A=1$, $B=1$

$K=\sqrt{1^2+1^2}=\sqrt{2}$

$$\sin x+\cos x=\frac{\sqrt{2}}{\sqrt{2}}\sin x+\frac{\sqrt{2}}{\sqrt{2}}\cos x$$

$$=\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right]$$

$$=\sqrt{2}\left[\frac{\sqrt{2}}{2}\sin x+\frac{\sqrt{2}}{2}\cos x\right]$$

$\cos\alpha=\frac{\sqrt{2}}{2}$ $\sin\alpha=\frac{\sqrt{2}}{2}$

$\alpha=\frac{\pi}{4}$

$$\sqrt{2}\left[\overset{A}{\sin x}\cdot\overset{B}{\cos\frac{\pi}{4}}+\overset{A}{\cos x}\cdot\overset{B}{\sin\frac{\pi}{4}}\right]$$

$$=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$$

Nov 14-11:07 AM

$$\underset{\substack{\uparrow\\ A=\sqrt{3}}}{\sqrt{3}}\sin x-\underset{\substack{\uparrow\\ B=-1}}{\cos x}$$

$K=\sqrt{A^2+B^2}$

$=\sqrt{(\sqrt{3})^2+(-1)^2}$

$=\sqrt{3+1}=\sqrt{4}=2$

$$2\cdot\frac{\sqrt{3}}{2}\sin x-2\cdot\frac{1}{2}\cos x=$$

$$2\left[\underset{\substack{\uparrow\\ \cos\alpha=\frac{\sqrt{3}}{2}}}{\frac{\sqrt{3}}{2}}\sin x-\underset{\substack{\uparrow\\ \sin\alpha=-\frac{1}{2}}}{\frac{1}{2}}\cos x\right]=2\left[\overset{A}{\sin x}\,\overset{B}{\cos\frac{11\pi}{6}}+\overset{A}{\cos x}\,\overset{B}{\sin\frac{11\pi}{6}}\right]$$

$=2\sin\left(x+\frac{11\pi}{6}\right)$

QIV RA. $30^\circ=\frac{\pi}{6}$

$\angle\alpha=2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$

Nov 14-11:14 AM

Solve

$$\sqrt{3}\ \sin x - \cos x = 1$$

$$K = 2$$

$$\boxed{\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x} = \frac{1}{2}$$

$$\sin x \cos\frac{11\pi}{6} + \cos x \sin\frac{11\pi}{6} = \frac{1}{2}$$

$$\sin\left(x + \frac{11\pi}{6}\right) = \frac{1}{2}$$

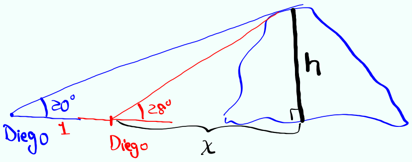
RA. 30°

Nov 14-11:20 AM

Diego's angle of elevation to the Top of Mt. Baldy is 20°.

He walked 1 mile Towards the Mt. Baldy, and angle of elevation became 28°.

How high is Mt. Baldy?



$$\tan 28° = \frac{h}{x} \qquad h = x \tan 28°$$

$$\tan 20° = \frac{h}{x+1} \qquad h = (x+1)\tan 20°$$

$$(x+1)\tan 20° = x \tan 28°$$

$$x \tan 20° + \tan 20° = x \tan 28°$$

$$\tan 20° = x(\tan 28° - \tan 20°) \qquad x = \frac{\tan 20°}{\tan 28° - \tan 20°}$$

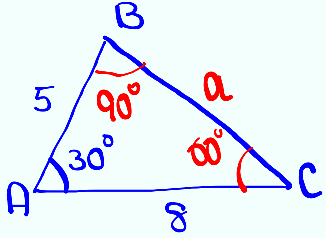
$$x \approx 2.2 \text{ miles}$$

$$h = 2.2 \tan 28°$$

$\approx \boxed{1.2 \text{ miles high}}$

Nov 14-11:23 AM

Solve the triangle below



$$a^2 = 5^2 + 8^2 - 2\cdot 5\cdot 8 \cos 30°$$

$$a^2 = 19.718$$

$$a = 4.4 \qquad \boxed{a \approx 4}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

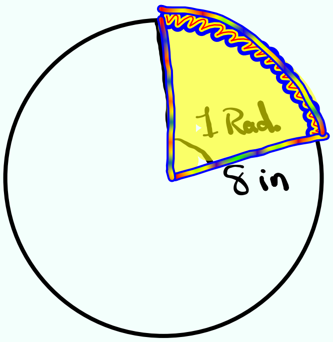
$$\frac{\overset{1/2}{\cancel{\sin 30°}}}{\underset{1}{\cancel{4}}} = \frac{\sin B}{\underset{2}{\cancel{8}}}$$

$$\sin B = 2\cdot\frac{1}{2} = 1$$

$$B \approx 90°$$

Nov 14-11:31 AM

Consider the circular sector below



1) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\cdot 8^2\cdot 1 = 32 \text{ in}^2$

2) $S = r\theta = 8\cdot 1 = 8$ in.

3) Perimeter of the sector.

P = 8 + 8 + 8 =
24 in.

Nov 14-11:37 AM

A blade of a fan makes 5 $\underbrace{\text{revolutions}}_{2\pi \text{ Rad.}}$ Per 8 Seconds.

Find angular velocity Rad./min.

$$\frac{5\cdot\cancel{2}\pi \text{ Rad}}{\cancel{8}_{4} \text{ Seconds}} \cdot \frac{\overset{15}{\cancel{60}} \text{ Seconds}}{1 \text{ Min}} = 75\pi \text{ Rad}/\text{min.}$$